Math 005C — Exam 3 Study Guide

This study guide is designed to help you prepare specifically for **Exam 3**. The topics below reflect the skills and concepts you should be ready to demonstrate. Focus on understanding *when* a particular coordinate system or technique is appropriate, not just how to carry out computations.

1. Iterated Integrals

What you should be able to do:

- Evaluate a double integral written as an iterated integral.
- Integrate with respect to one variable while treating the other as constant.
- Carefully apply limits of integration in the correct order.
- Simplify exponential expressions when possible before integrating.

You should understand:

- The order of integration matters in the setup.
- Inner limits correspond to the inner variable.
- Outer limits correspond to the outer variable.

2. Center of Mass of a Lamina

What you should be able to do:

- Identify an efficient coordinate system for a planar region.
- Express density as a function of position.
- Set up the integral for total mass.
- Set up the appropriate moment integral needed to find a coordinate of the center of mass.

You should understand:

- Density proportional to distance from an axis or origin must be written as a function.
- Polar coordinates are often best for circular regions.

3. Reversing the Order of Integration

What you should be able to do:

- Sketch the region described by bounding curves.
- Write the same double integral in both orders: $dy\,dx$ and $dx\,dy$.
- Determine which order is easier to evaluate.
- Compute the integral using either valid setup.

4. Multiple Integrals

What you should be able to do:

- Identify upper and lower surfaces of a solid.
- Choose an appropriate coordinate system.
- Set up a rectangular-coordinate double integral.
- Set up a polar-coordinate double integral.
- Set up a rectangular-coordinate triple integral.
- Set up a cylindrical-coordinate triple integral.
- Set up a spherical-coordinate triple integral.
- Recall what dA and dV are, as appropriate, in all each coordinate system.

Practice ideas for volume setup:

1. Set up the volume enclosed by $z = \sqrt{x^2 + y^2}$ and $z = 3$.
2. Set up the volume enclosed by $z = x^2 + y^2$ and $z = 4$.
3. Set up the volume above $z = 1$ inside the sphere $x^2 + y^2 + z^2 = 9$.

5. Change of Variables in Double Integrals

What you should be able to do:

- Apply a given transformation.
- Rewrite the boundary curves in the new variables.
- Find the Jacobian.
- Rewrite the integrand in terms of the new variables.
- Evaluate the transformed integral.

Common mistakes to avoid

- Forgetting the Jacobian in polar, cylindrical, or spherical coordinates.
- Reversing integration bounds incorrectly.
- Using the wrong projection region.
- Forgetting top minus bottom in volume problems.
- Using incorrect conversions for dA and dV
- Omitting absolute value in a Jacobian.

Practice Worksheet

1. Evaluate: $\int_0^1 \int_0^2 xye^x \, dy \, dx$
2. A lamina occupies the upper half of the disk $x^2 + y^2 \leq 9$ with density proportional to the distance from the origin. Set up the integrals needed to find the mass and center of mass.
3. Let D be the region bounded by $x = y^2$ and $x = 4y$. Set up $\iint_D (x + y) \, dA$ in both orders of integration ($dy \, dx$ and $dx \, dy$), then compute using either order.
4. Set up a cylindrical-coordinate integral for the volume inside $z = 4 - x^2 - y^2$ and above the xy-plane.

5. Write a rectangular-coordinate double integral for the volume under $z = 6 - \sqrt{x^2 + y^2}$ over the disk $x^2 + y^2 \leq 9$.

6. Evaluate $\iiint_E (x^2 + y^2 + z^2)^{3/2}\, dV$,where E is the solid ball $x^2 + y^2 + z^2 \leq 1$ in the first octant.

7. Use the transformation $u = 2x + y$, $v = x - 2y$ to evaluate $\iint_R (2x + y)(x - 2y)\, dA$,where R is the rectangular region enclosed by the lines $2x + y = 0$, $2x + y = 3$, $x - 2y = 1$, and $x - 2y = 4$.